the experimentalist using an LV, the Basset integral term may be neglected since C is small compared to the other coefficients in Eq. (1). Lastly, the lift force is incorporated into the coefficient D, only for generality.

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Comment on "Buckling of Composite Plates with a Free Edge in Edgewise Bending and Compression"

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TABLE 2 of Ref. 1 presented buckling loads for a laminated rectangular plate with two loaded edges simply supported, one side free and the opposite side either simply supported or clamped, for linearly varying edge loads. Since the analysis of Ref. 1 is based on classical plate theory, it is of interest to compare those results with buckling loads that include the effect of transverse shear deformations.

Such buckling loads have been presented for plates with uniformly loaded edges.[2] These were calculated using the shell-of-revolution program FASOR[3] by considering the plate as part of a cylinder of very large radius. The circumferential direction of the cylindrical model corresponds to the longitudinal (loaded) direction of the plate, and the axial length of the cylinder equals the plate width b. The plate buckling load is obtained by minimizing the model buckling load with respect to the circumferential wave number N subject to the condition $N = n\pi R/a$, where n is the number of longitudinal half-waves of the plate buckle, R is the radius of the cylindrical model, and a is the length of the plate. This procedure is also applicable to plates with linearly varying edge loading.

The dimensions of the laminated plate analyzed in Ref. 1 are $a = 254$ mm (10 in.) and $b = 50.8$ mm (2 in.). It is made of 0.127-mm (0.005-in.)-thick tape with the following in-surface elastic properties:[4] $E_1/E_2 = 10.05$, $G_{12}/E_2 = 0.349$, $\mu_1 = 0.34$, and $E_2 = 13.03$ GPa (1.89×10^6 psi), where 1 and 2 signify directions parallel and transverse, respectively, to the fibers and μ_1 is the major Poisson's ratio. The laminate definition is $[\pm 45_3/0_3]_s$ (erroneously reported as $[\pm 45_3/0_3/90_3]_s$ in Ref. 1), thus giving a laminate thickness $h = 2.286$ mm (0.090 in.) Neglecting the small anisotropic effect, this laminate has the bending stiffness matrix given by Eq. (25) of Ref. 1.

Since the values of the transverse shear moduli G_{13} and G_{23} are unavailable, it is assumed that $G_{13} = G_{23} = G_{12}$. (For typical unidirectional laminae, $G_{23} < G_{12}$.) Again neglecting the small anisotropic effect, FASOR gives the transverse shear stiffness matrix[5] $[K] = 8.951 \times 10^6$ N/m (5.112×10^4 lb/in.) $[I]$, where $[I]$ is the 2×2 unit matrix.

Table 1 compares the classical plate theory results of Ref. 1 with the transverse shear deformation theory results calculated by FASOR. It is noted that the buckling load knockdowns due to transverse shear deformation for this plate are smaller than those reported for uniformly compressed plates in Ref. 2. Aside from differences in laminar properties, lay-up, and edge conditions, this is not surprising since the thickness-to-width ratio for this plate is $h/b = 0.045$, which is less than one-half that of the thinnest plate studied in Ref. 2.

Table 1 Buckling stress resultants $N_x b^2/E_2 h^3$ at free edge ($y = b$)

	Condition at supported edge ($y = 0$)					
	Simply supported			Clamped		
$N_x(0)/N_x(b)$	CPT[a]	FASOR[b]	Diff., %	CPT[a]	FASOR[b]	Diff., %
100	0.101	0.097	4.0	0.219	0.202	8.6
2	2.106	2.018	4.4	4.221	3.954	6.8
1	2.637	2.525	4.4	5.133	4.792	7.1
0.5	3.016	2.886	4.5	5.745	5.357	7.2
0.01	3.507	3.356	4.5	6.491	6.042	7.4
−0.5	4.221	4.036	4.6	7.487	6.952	7.7
−1	5.263	5.027	4.7	8.762	8.121	7.9
−2	10.033	9.579	4.7	12.777	11.744	8.8

[a]Classical plate theory (Ref. 1). [b]Transverse shear deformation theory.

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Comment on "Stiffness Matrix Adjustment Using Mode Data"

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THE above paper[1] is a welcome addition to the literatures of structural system identification. The author presents a stiffness matrix adjustment (KMA) procedure that shows some promising results. Nevertheless, we believe that the following comments are appropriate for further developments in the field, especially in relation to real large structures.

Let p be the number of constraints (equations) and q the number of unknowns to be identified. Depending on relative values of p and q, there are essentially two kinds of system identification.[2,3] For an overdetermined system, $p > q$, a least-square solution is sought that minimizes errors of the solution. The procedures given in Refs. 4 and 5 fall into this category. On the other hand, if $p < q$ (underdetermined system), there are infinitely many sets of solutions satisfying given con-

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straints. Among them, the accepted solution usually involves the minimization of some specified functions. References 1, 6, and 7 are based on this minimum-norm approach.

We shall first discuss a philosophical point of considerable significance. In a real application, the "correct" model is unknown. For the underdetermined class of problems (including that of Ref. 1), the changes in a baseline model, satisfying all constraints, are minimized by some criteria. Since there are an infinite number of solutions that satisfy all the constraints, there is no particular reason to believe the minimum changed model is "correct" unless the analyst has confidence in the baseline (analytical) model. When the minimum changes are "small," in the analyst's judgement, the modified model may be considered to be "better," i.e., actually an improved analytical model. The small changes also enhance an analyst's confidence in the original model. In the example presented in Ref. 1, changes of about 40% were required. In a practical case (where the "correct" answer is unknown), the analyst would have no confidence in the resulting model, and he would be forced to conclude that the original analysis or the test data or both contained significant errors. These and other closely related issues were discussed in more detail in Ref. 8. Another related issue is the statement in Ref. 1 that increasing the ratio of p/q results in "more accurate identification." This is somewhat a matter of definition, however, when the exact model is unknown. Adding more measured modes increases the changes from the known baseline and makes the results more sensitive to errors in the test data. The greater changes in the analytical model reduces the analyst's confidence in the results.

It is recognized that the above philosophical points may be controversial. The following discussion will concentrate on technical problems.

The greatest strength of KMA is the preservation of structural connectivity, and the greatest weakness is the very large amount of computation it must perform. Analysis of Ref. 1 results in $n \times m$ algebraic equations [n the number of degrees of freedom (DOF) and m the number of test modes] with the same number of unknowns (Lagrange multipliers). An eigenanalysis of a matrix of order $n \times m$ is then invoked to solve for these unknowns. It is not unusual in practice to identify a structural system having several hundred DOF or more using 10-20 or more test modes. Eigenanalysis on systems of the order of thousands are required. For realistic structural models, KMA is highly inefficient, if not actually impossible. Note that there are other identification procedures[6,7] that formulate the identified model as series of basic matrix operations (addition and multiplication of matrix of dimension up to $n \times n$, and inversion of a matrix of rank m) of analytical model and test modes and avoid numerical solutions of large algebraic systems. For example, in Ref. 7, the analytical model improvement (AMI) procedure is successfully applied to identify a model of 508 DOF using 19 test modes. KMA is obviously overextended when treating a system of this size. It should also be pointed out that a similar procedure, also preserving the structural configuration, was proposed to identify the mass matrix by Berman and Flannelly[9] in 1971. This method was not pursued because of the intractable amount of computations it involved for realistic-sized problems.

It is obvious that the preservation of structural connectivity achieved by KMA is due to its element-by-element improvement of the stiffness matrix, which in turn allows definition of ϵ, the norm to be minimized, in terms of percentage changes. We agree on the importance of minimization of percentage changes (rather than absolute magnitudes), which prevents unrealistic changes as described in Ref. 1. Actually the weighting matrices in Refs. 2, 6, and 7 were, in a sense, introduced for this purpose. If the norms for identifying M and K are weighted by their analytical counterparts, $M_A^{-1/2}$ [Eq. (5) of Ref. 7] and $K_A^{-1/2}$ [Eq. (14) of Ref. 2], respectively, then biased changes can be controlled to a certain extent. The KMA definition of ϵ, in fact, is not novel. In his 1979 Comment,[10] one of us discussed the same problem and suggested the same percent norm. This approach was also not pursued because using this norm results in a large algebraic system where a closed-form solution for identified model, as given in Ref. 2, 6, and 7, is impossible. Therefore, the beneficial properties of KMA (preserving structural configuration and the simplicity of defining a percentage norm) and its disadvantage (processing a matrix of order $n \times m$) both spring from the same source: element-by-element improvement.

To relieve the overwhelming amount of computation, Ref. 1 suggests removing weak coupling terms by dynamic model condensation and then applying KMA to identify the stiffness matrix while keeping the modified structural configuration unchanged. Since major reduction in DOF in model condensation yields fully populated matrices, the recommended preprocessing requires eliminating weak couplings to make the problem solvable. This implies that, in practical applications, KMA does not really preserve the structural connectivity and has only a limited advantage over the other identification procedures in this respect.

In summary, we conclude that a system identification procedure, to be applicable to actual structures, should avoid biased changes, preserve structural connectivity specified by the user, but not invoke element-by-element improvement.

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